4th YSM-SPIP in Sendai/Prologue Series IV of FSPIP

Duality with real-space renormalization and its application to bond percolation

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We rederive the exact solution for the bond-percolation thresholds in the inhomogenous case on the square lattice by use of the duality with real-space renormalization, which is inspired by the star-triangle transformation [1]. In addition, we obtain two different generic formulas on the square lattice depending on the tiling manner of the unit cell to cover the whole lattice. Both equalities can be reduced to the known formula for the triangle cell, which includes the triangular and hexagonal lattices [2]. The application of the generic formula on the square lattice reproduces the exact solution on the bow-tie lattice and the well-known approximate solution on the Kagomé lattice [3, 4].

The duality analysis with real-space renormalization is essentially the same as the special technique, which has developed in context of spin glass theory. The method is also useful to describe the precise phase boundary [5, 6]. Then we need systematic renormalization to enhance precision of estimations by taking a relatively larger size of the cluster with many bonds than the unit cell we dealt with in the present study.

The exact solutions for finite dimensional many-body systems have been rare in spite of the long-year efforts. However the present study implies that the situation begins to change by development of the duality analysis, which is found to be applicable to a relatively broad class of problems, namely spin glasses and inhomogenous percolation problems.

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