Hierarchical Noise-Intensity Fluctuations in Langevin Modeling

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abstract

Langevin models have become increasingly important in modeling systems subject to fluctuations. In general, fluctuations are space- and time-dependent phenomena; hence, the noise intensity fluctuates temporally and/or spatially. We considers a Langevin model, where the noise-intensity is governed by the Ornstein–Uhlenbeck process [1]:

$$\dot{x} = f(x) + g(x)s\xi_x(t); \quad \dot{s} = -\gamma(s-\alpha) + \sqrt{\gamma}\xi_s(t), \tag{1}$$

where α denotes the mean of $s(t)$, and $\xi_x(t)$ and $\xi_s(t)$ are white Gaussian noise with correlation functions $\langle \xi_x(t)\xi_x(t')\rangle = 2D_x\delta(t-t')$ and $\langle \xi_s(t)\xi_s(t')\rangle = 2D_s\delta(t-t')$ (D_x and D_s are the noise-intensity). In Eq. (1), we call a term $s\xi_x(t)$ *stochastic intensity noise* (SIN), because the noise-intensity is modulated by a stochastic process. The kurtosis of SIN is $\kappa = 9 - 6/(1+\rho)^2$, where $\rho = D_s/\alpha^2$ represents the squared variation coefficient, ratio between the variance and the squared mean in Eq. (1). Figure 1 shows examples of time course of SIN [(a) and (b)] as well as their histograms [(c) and (d)].

Two dimentional Fokker–Planck equation (FPE) of Eq. (1) is given by $\partial_t P(x,s;t) = \mathrm{L_{FP}}P(x,s;t)$, where $\mathrm{L_{FP}}$ is an FPE operator. We developed an approximation scheme for the FPE, which casts the two dimentional equation into one dimentional equation in terms of x, by using the adiabatic elimination. The obtained equation is given by [1]

$$\partial_t P(x;t) = \left[-\partial_x f(x) + \left\{D_x(D_s+\alpha^2)\right\}\triangle_g + \left\{(D_x^2 D_s(4\alpha^2+D_s))/\gamma\right\}\triangle_g^2\right] P(x;t), \tag{2}$$

with $\triangle_g = \partial_x^2 g(x)^2 - \partial_x g'(x)g(x)$. Although Eq. (2) includes higher-order derivatives than the second, we have solved Eq. (2) with perturbation expansion and showed that it can be applied to several nonlinear systems including a gene expression mechanism [1].

We also investigated statistical properties of systems driven by SIN, specifically in bistable and ratchet potentials [1] with matrix continued fraction method. We calculated the mean first passage time and stochastic resonance in the bistable potential, and the current in the ratchet potentials.

References

[1] Y. Hasegawa and M. Arita, Physica A, 389, 4450 (2010); Y. Hasegawa and M. Arita, Physica A, 390, 1051 (2011); Y. Hasegawa and M. Arita, Phys. Lett. A, 375, 3450 (2011); Y. Hasegawa and M. Arita, arXiv:1112.5287.

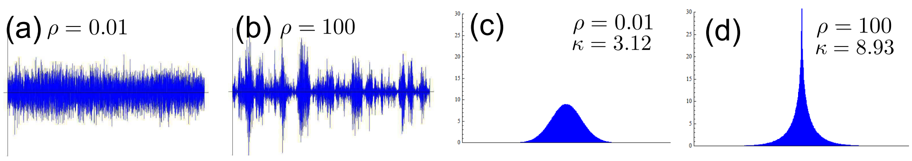

Figure 1: (a) and (b) Time course of SIN created by MC simulations. (c) and (d) Histograms of SIN. ρ and κ denote the squared variation coefficient and the kurtosis, respectively.