量子アニーリングが拓く新時代

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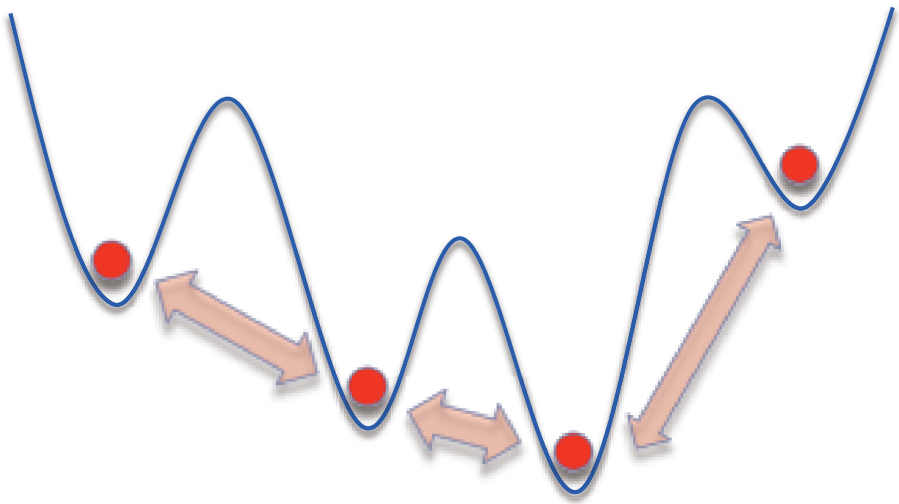
基盤研究 (B)「量子アニーリングが拓く機械学習と計算技術の新時代」
H.27.4.1〜 with 田中宗（早稲田大)、安田宗樹（山形大)、中島千尋（東北大)

Contents

History of quantum annealing

- Simulated annealing
  S. Kirkpatrick, et al, Science 220 (1983) 671.
- Quantum annealing
  T. Kadowaki and H. Nishimori, Phys. Rev. E 58 (1998) 5355.
- Quantum adiabatic computation
  E. Farhi et al, Science 292 (2001) 472.



What is quantum power?

It is capable to efficiently solve the optimization problem.
The factorization problem (NP?) can be solved by quantum computer.
[Shor's factorization problem 1994]

Architecture of quantum computer is proposed in two ways:

- Quantum gate
  - Sequential application of unitary gates
  - Tricky algorithm depending on optimization problem
- Quantum annealing
  - Just only cooling the system
  - A simple algorithm for any optimization problem

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Quantum annealing

By solving Shrödingier eq. (direct manipulation in nature),

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = H(t)\Psi(t), \tag{1}$$

where

$$H(t) = \frac{t}{T}H_0 + \left(1 - \frac{t}{T}\right)H_1. \tag{2}$$

Basic formulation

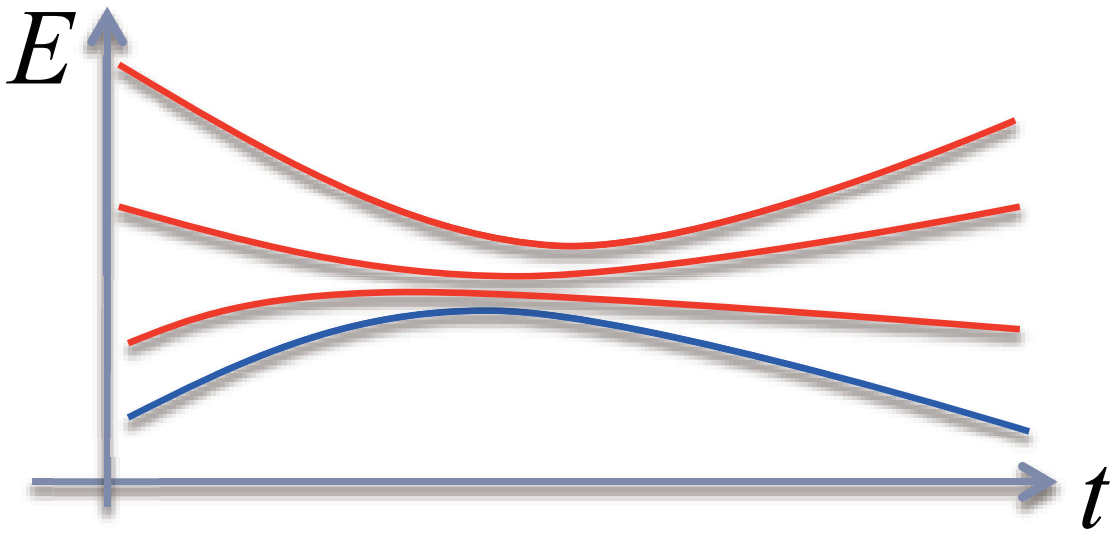
- Cost function of optimization problem $= H_0$
- Driver Hamiltonian $= H_1$

Adiabatic evolution

- Initial state is the ground state for H_1.
  Slow driving ensures that
- Final state is the ground state for H_0.

Instantaneous Hamiltonian is

$$H(t) = \frac{t}{T} H_0 + \left(1 - \frac{t}{T}\right) H_1. \tag{3}$$



Rough sketch: one spin in transverse field

$$\frac{t}{T}H_0 + \left(1-\frac{t}{T}\right)(-\Gamma\sigma_1^x) = \begin{pmatrix} \frac{t}{T}H_0(+) & -\Gamma\left(1-\frac{t}{T}\right) \\ -\Gamma\left(1-\frac{t}{T}\right) & \frac{t}{T}H_0(-) \end{pmatrix}. \quad (4)$$

The non-diagonal elements express the hopping between states.

Adiabatic theorem

Adiabatic theorem demands, for $P(T) = |\Psi(t)|^2 \approx 1 - \epsilon$,

$$T_{\text{an.}} \approx \frac{1}{\epsilon \min_t \Delta(t)^2} \tag{5}$$

Residual energy is estimated as $E_{\text{res}} = 1/T^2$ where T is a computation time.
[S. Suzuki and M. Okada (2005)]

Phase transition

In infinite-size system, the phase transition takes place at a some point

	Quantum	time in QA
1st-order trans.	$\Delta \sim \alpha^{-N}$	$T_{\mathrm{an.}} \sim \alpha^{2N}$
2nd-order trans.	$\Delta \sim N^{-\beta}$	$T_{\mathrm{an.}} \sim N^{2\beta}$

Analysis done by diagonalization and Monte-Carlo simulation.

Roughly speaking, quantum annealing for optimization problems involves

- P: second-order transition
- NP: first-order transition

Quantum annealing has the same performance as the classical computer?

Where is the boundary between Quantum and Classical?

Quantum Classical mapping (Nishimori et. al. (2014)

(imaginary) Quantum system can not always be mapped to classical system

$$\frac{d}{dt}\Psi(t) = H(t)\Psi(t) \leftrightarrow ? \frac{dP(t)}{dt} = W(t)P(t) \tag{6}$$

- (diagonal part) Remaining Prob. in SA = (-)cost func. in QA
- (non-diagonal) hopping Prob. in SA = (-)driver Hamiltonian in QA

QA with (non-diag.) negative elements	$\Rightarrow$	SA
QA with (non-diag.) positive elements	$\Rightarrow$	No classical algorithm

Future direction 1
Beyond classical computer!

Positive non-diagonal elements

Another driver Hamiltonian is used to induce the other type of quantum fluctuation

$$H(t) = \lambda \{sH_0 + (1-s)H_1\} + (1-\lambda)H_2 \tag{7}$$

where

$$H_2 = +\Gamma' \left(\sum_i \sigma_i^x \right)^2 \tag{8}$$

[Y. Seki and H. Nishimori (2012)]

Not authorized figures!

Future direction 2
Why do you stand on the ground state?

Non-adiabatic quantum annealing

Starting from ensemble with excited states under thermal fluctuations.
Perform the relatively high-speed annealing.

- Quantum Jarzynski annealing [M. Ohzeki, PRL (2010)]
- Nonadiabatic quantum annealing [M. Ohzeki, et al. JPSJ (2011)]
- Experimental study [N. G. Dickson et al. Nature comm. (2013)]

From top to bottom, the time for QA is shortened.

Not authorized figures!

Non-adiabatic quantum annealing (Somma et al. (2012)

The oracular problem.

- Sub-exp order in classical computation.
- An efficient method based on the quantum walk exists.

In QA, closure of the energy gap appears twice, and then the intermediate state is an excited state but the final ground state is achievable.

Not authorized figures!

Is efficient quantum speedup based on the multiple transition?

Conclusion

Quantum annealing from a point of view of computational cost

- Negative non-diagonal elements: classically simulatable
- Positive non-diagonal elements: not classically simulatable

Two future directions

- QA with nontrivial driver Hamiltonian
- Multiple transition between ground state and excited state
  Do not stick to the ground state while QA!

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