

Localizing chaotic needles in regular haystacks (and conversely) using Lyapunov Weighted Dynamics.

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The transition from order to chaos has been a major subject of research since the work of Poincare, as it is relevant in areas ranging from the foundations of statistical physics to the stability of the solar system. Along this transition, atypical structures like the first chaotic regions to appear, or the last regular islands to survive, play a crucial role in many physical situations. For instance, resonances and separatrices determine the fate of planetary systems, and localised objects like solitons and breathers provide mechanisms of energy transport in nonlinear systems such as Bose-Einstein condensates and biological molecules.

Unfortunately, despite the fundamental progress made in the last years, most of the numerical methods to locate these ‘rare’ trajectories are confined to low-dimensional or toy models, while the realms of statistical physics, chemical reactions, or astronomy are still hard to reach.

I will show how one can use the Lyapunov Weighted Dynamics [1] to sample the large deviations of Lyapunov exponents, whence selecting trajectories with unusual chaoticity. After presenting the algorithm and the underlying theory, I will show on a number of examples [2], ranging from astronomy-oriented classical mechanics problems to chains of oscillators or coupled maps that belong to the realm statistical mechanics, how one can single out atypical trajectories of desired chaoticities.

Beyond the numerical detection of atypical trajectories, I will show how the algorithm allows one to compute the topological pressure of extended dynamical systems [2], a central quantity in the Thermodynamic of Trajectories of Ruelle. This observable plays the role of a free energy in trajectory space and thus provide a natural framework to discuss dynamical phase transition.

[1] J. Tailleur, J. Kurchan, “Probing rare physical trajectories with Lyapunov-Weighted Dynamics”, *Nature Physics*, 3, p. 203-207, (2007)

[2] T. Laffargue, K.-D. Nguyen Thu Lam, J. Kurchan, J. Tailleur, “Large deviations of Lyapunov exponents”, arxiv:1302.6254