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Information Thermodynamics on Causal Networks

Sosuke Ito[1] and Takahiro Sagawa[2]
1. Department of Physics, The University of Tokyo
2. Department of Basic Science, The University of Tokyo

Nonequlibrium relations for small thermodynamic systems such as molecular motors have been intensively investigated in these decades. The second law of thermodynamics (*i.e.*, $\langle \sigma \rangle \geq 0$) can be derived from the Jarzynski equality (or the integrated fluctuation theorem):

$$\langle \exp[-\sigma] \rangle = 1, \tag{1}$$

where σ is the stochastic entropy production and $\langle \cdots \rangle$ describes the ensemble average.

On the other hand, in the presence of feedback control by Maxwell demon, the second law seems to be violated, *i.e.*, $\langle \sigma \rangle$ can be negative. For such cases, we have a generalized second law $\langle \sigma \rangle \geq \langle \Delta I \rangle$ and a generalized Jarzynski equality

$$\langle \exp[-\sigma + \Delta I] \rangle = 1, \tag{2}$$

where $\langle \Delta I \rangle$ is mutual information that is exchanged between the system and the feedback controller [1,2]. Although the relations are applicable to nonequilibrium dynamics with a single information exchange, the general theory has been elusive for more complex cases, in which multiple systems exchange information many times.

Here, we study a system that is involved in a complex information flow induced by multiple other systems. Characterizing the interaction of multiple systems by a causal network (*i.e.*, Bayesian network such as Fig. 1), we obtain a new generalization of the Jarzynski equality

$$\langle \exp[-\sigma + \Theta] \rangle = 1, \tag{3}$$

which leads to a new generalized second law $\langle \sigma \rangle \geq \langle \Theta \rangle$. Here, $\langle \Theta \rangle$ is a quantity that consists of the transfer entropy [3] and the exchanged mutual information between multiple systems. For special cases, Eq. (3) reduces to Eqs. (1) and (2) where the dynamics are characterized by Figs. 2 and 3, respectively.

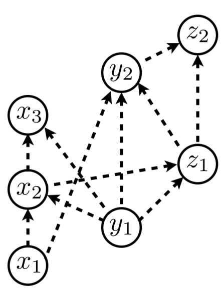

Figure 1: An example of the Bayesian network.

Figure 2: A Bayesian network that describes the Markov process.

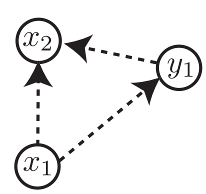

Figure 3: A Bayesian network with a feedback controller described by y_1.

[1] T. Sagawa and M. Ueda, Phys. Rev. Lett. **104**, 090602 (2010).
[2] T. Sagawa and M. Ueda, Phys. Rev. Lett. **109**, 180602 (2012).
[3] T. Schreiber, Phys. Rev. Lett. **85**, 461 (2000).