Frontier of Statistical Physics and Information Processing 2013

Reliable Cellular Automaton in One Dimension

Keisuke FUJII[1,2],

1. The Hakubi Center for Advanced Research, Kyoto University
2. Graduate School of Informatics, Kyoto University

It has been known that no finite-temperature phase transition occurs in one-dimensional (1D) equilibrium systems with finite-range interactions [1]. The positive rates conjecture is the generalization of the above claim to 1D probabilistic cellular automata, whose neighborhoods are of finite range. That is, the positive rates conjecture asserts that information cannot be stored reliably in 1D probabilistic cellular automata in the presence of errors of positive rates. While the conjecture has been believed to be true for 30 years, it was refuted by P. Gács, who constructed a reliable probabilistic automaton in 1D, which utilized 2^{100} states in each site and a complicated update rule in order to perform self-organization [2].

Here we propose another reliable probabilistic cellular automaton in 1D, which consists of only 4 states in each site and a simple update rule. Specifically, the nonequilibrium dynamics of the 1D probabilistic cellular automaton is mapped into an equilibrium configuration of a two-dimensional random-bond Ising model (RBIM). This mapping allows us to bound the logical error probability rigorously by calculating the domain-boundary free energy of the RBIM. It has been shown that if the physical error probability is sufficiently smaller than a constant value, the logical error probability decreases exponentially in the system size.

[1] T. Liggett, *Interacting Particle Systems*, Springer-Verlag, New York, 1985.
[2] P. Gács, J. Stat. Phys. **103**, 45 (2001).