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Universal implementation of energy eigenbasis measurement

Shojun NAKAYAMA[1], Akihito SOEDA[1,2]A, and Mio MURAO[1,3]

1. Department of Physics, Graduate School of Science, the University of Tokyo
2. Centre for Quantum Technologies, National University of Singapore
3. Institute for Nano Quantum Information Electronics, the University of Tokyo

We show a scheme to universally implement a projective measurement in the energy eigenbasis on a system evolving by an unknown Hamiltonian H based on the phase estimation algorithm. One proposal assumes that the input is encoded in a particular subspace and that there is another subspace on which the Hamiltonian acts as the identity operator[1]. In this case, the phase estimation algorithm can be applied without estimating the Hamiltonian. The assumptions are satisfied in particular setups such as in linear optical quantum computation using photon qubits, but cannot be generally applied to other settings.

To apply the phase estimation algorithms for unknown Hamiltonian systems, two new algorithms are introduced. One is for asymptotically but universally implementing a controlled-unitary operation $C_{U(t)}$ of a unitary operation $U(t) = e^{-iHt}$ up to the global phase of $U(t)$ for an unknown Hamiltonian H using dynamical decoupling[2]. Another is a new deterministic quantum computation with one pure qubit (DQC1) algorithm[3] for evaluating the absolute value of the trace of $U(t)$ without using $C_{U(t)}$. We analyze accuracy of the implementation of $C_{U(t)}$ by using this DQC1 algorithm. The DQC1 algorithm is also used for removing the ambiguity due to the periodicity of the phase and the effect of the global phase to obtain the energy eigenvalue as an outcome. Maximally mixed states can be used as ancilla systems in both algorithms.

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