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Anomalous System Size Dependence of Large Deviation Functions for Local Empirical Measure

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In this poster presentation, we study the large deviation functions for a local empirical measure (the empirical measure at one fixed position) in systems with translational symmetry. Such physical quantities are seen, for example, in the scenario of observing a density or staying time of diffusing particles at one fixed position in laboratory experiments. The local empirical measure at a position $\boldsymbol{x} \in \mathbb{R}^d$ is defined as

$$\mu_{\boldsymbol{x}}(\tau) := \frac{1}{\tau}\int_0^\tau \rho_{\boldsymbol{x}}(t)dt, \tag{1}$$

where $\rho_{\boldsymbol{x}}(t)$ is a fluctuating density at $\boldsymbol{x}$ and at time t. The large deviation function for $\mu_{\boldsymbol{x}}(\tau)$ is denoted by $I(\mu)$. That is,

$$\mathrm{Prob}(\mu_{\boldsymbol{x}}(\tau) = \mu) \sim e^{-\tau I(\mu)}. \tag{2}$$

Here, "$A \sim B$" indicates $\lim_{\tau\to\infty} \ln A/\tau = \lim_{\tau\to\infty} \ln B/\tau$.

We find that $I(\mu)$ shows anomalous dependence on system size. Naively, we expect that $I(\mu)$ converges in the thermodynamic limit because $I(\mu)$ is a local quantity. Thus, $I(\mu)$ seems to take a finite value in the thermodynamic limit. For systems in more than and equal to three dimensions, such a naive expectation holds true. Surprisingly, however, the naive expectation fails for systems in one or two dimensions. Specifically, in the thermodynamic limit, the variance of $\mu_{\boldsymbol{x}}(\tau)$, which can be calculated from $I(\mu)$, diverges in proportion to system size L in one dimension, and $\ln L$ in two dimensions [1].

First, in order to grasp the properties of this anomaly, we analyze a solvable microscopic model, specifically independent random walks on a lattice with discrete translational symmetry. Through this analysis, we find that the necessary conditions for this anomaly are (i) there exists no macroscopic flow, and (ii) their space dimension is one or two. Second, we treat systems whose distribution functions follow the Fokker-Planck equation. We derive this anomaly by applying a contraction principle [2] to the formula of the large deviation function for the empirical distribution function [3].

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