Frontier of Statistical Physics and Information Processing 2013

A relation between replica symmetry and a performance of approximation algorithms in minimum vertex cover problems on uniform random hypergraphs

Satoshi TAKABE and Koji HUKUSHIMA
Graduate school of Arts and Sciences, The University of Tokyo

Typical-case behavior of a class of randomized combinatorial optimization problems has attracted a great deal of interest in computer science and also practical aspects in human society. In this presentation we study minimum vertex cover problems (min-VCs) on random α-uniform hypergraphs using two different approaches [1], a replica method in statistical mechanics of random systems and a leaf removal algorithm, one of approximation algorithms for min-VCs. The random hypergraph has the same edge degree α and its vertex degree follows Poisson distribution. We find that there exists a phase transition at the critical average degree $e/(\alpha-1)$. Below the critical degree, a replica symmetric ansatz in the statistical-mechanical method holds and the algorithm estimates a solution of the problem which coincides with that by the replica method. In contrast, above the critical degree, the replica symmetric solution becomes unstable and these methods fail to estimate the exact solution. These results strongly suggest a close relation between the replica symmetry and the performance of the approximation algorithm.

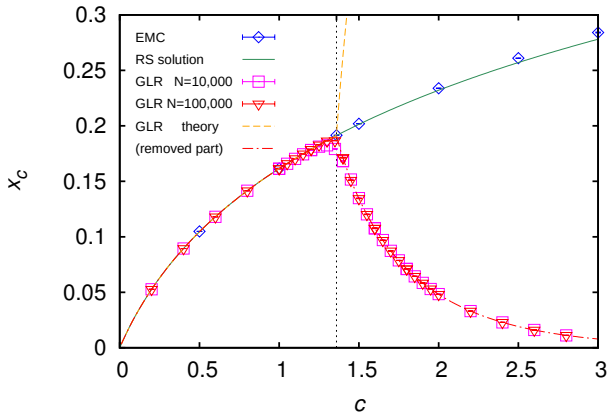

Figure 1: The average minimum-cover ratio on $\alpha = 3$-uniform random hypergraphs as a function of the average degree c. Open marks are numerical results by the exchange MC (diamonds) and by the leaf removal algorithm (squares and triangles). Lines represent analytical results by the replica method (solid), by the leaf removal algorithm (dashed) and on the removed part of the graphs by the algorithm (dashed-dotted). The vertical dotted line is the critical average degree $c^* = e/2$, below which all lines merge into a single line.

[1] S. Takabe and K. Hukushima arXiv:1301.5769.