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Markov Chain Monte Carlo Method with Skew Detailed Balance Condition

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The Markov chain Monte Carlo methods is a powerful tool for sampling from a high-dimensional probability distribution and for estimating an expectation value under the distribution. Although the detailed balance condition is imposed in practice in order to guarantee the method to work correctly, it is not always necessary. Several methods without the detailed balance condition have been proposed recently [1, 2]. We construct a Markov chain Monte Carlo method for a finite system of discrete degree of freedom on a basis of skew detailed balance condition [2]. In this method, an additive variable $\varepsilon \in \{+, -\}$ is introduced to the state space $I = \{1, 2, \ldots, S\}$. The skew detailed balance condition is imposed, where stochastic flow with a transition from a state i to j under $\varepsilon = +$ balances with the reverse process under $\varepsilon = -$. In addition, the total balance condition is satisfied by introducing a process for ε flip. We apply the method to a one-dimensional kinetic Ising model in order to understand the properties of the method [3]. We determine explicitly several transition probabilities which satisfy the skew detailed balance condition and evaluate analytically the time evolution of the magnetization density for a specific transition probability in a thermodynamic limit. As a result, we theoretically find that the relaxation time of magnetization density is reduced by breaking the detailed balance condition and that one of the transition probabilities even changes the dynamical critical exponent.

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