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Estimation of a Parameter in a Quantum Relaxation Process via a Single Sequence of Measurements without State Re-Initialization

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Estimating parameters of quantum systems (e.g., a parameter contained in the Hamiltonian of a quantum system) is important from both practical and fundamental aspects. In the standard strategies, parameters are estimated from the data accumulated by many independent and identical experiments: every time one performs an experiment, the system needs to be reset in a specific initial state. In contrast, we are going to discuss a scheme in which **one is not required to re-initialize the system after every measurement**: we simply repeat measurements ("sequential scheme"), and **a parameter is estimated from a *single* sequence of measurements**. It has been shown in Ref. [1] that the probability distribution of the average of the outcomes of a single sequence of measurements in a mixing Markov chain becomes approximated by a narrow Gaussian asymptotically, which ensures the estimability of a parameter from the single run. We are going to show that **the correlations among the measurement data can be useful in the sequential scheme for enhancing the precision of the estimation, going beyond the precisions by standard strategies**.

As an example, we consider the problem of estimating the temperature of a thermal bath. We put a probe qubit in contact with the bath and let it relax from an arbitrary initial state. During the evolution, we measure the probe at time intervals τ to check whether it is in the excited state $|0\rangle$ or in the ground state $|1\rangle$, obtaining a single sequence of measurement outcomes $\{s_1, \ldots, s_N\}$ ($s_i = 0, 1$ for $i = 1, \ldots, N$). **Neither the re-initialization of the probe state after every measurement nor the repetition of the experiment is required.** The measurements can be weak and unsharp: we consider a measurement model based on a CNOT gate, which contains a parameter η characterizes the strength and the precision of the measurement. We then evaluate the average $S = \frac{1}{N}\sum_{i=1}^{N} s_i$ and the correlations $C_\ell = \frac{1}{N-\ell}\sum_{i=1}^{N-\ell} s_i s_{i+\ell}$, from which we try to estimate the temperature of the bath.

We prove that the central limit theorem holds for the set of variables $(S, C_1, C_2, \ldots)$ asymptotically as the number of the measurements N increases, due to the mixing property of the relevant quantum channel $\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_1$ with $\mathcal{E}_s = \mathcal{M}_s \circ \Lambda_\tau$, where Λ_τ describes the time evolution of the probe under the influence of the bath for time τ (the solution to a master equation) and $\mathcal{M}_s$ represents the effect of the measurement on the probe when the measurement result $s\,(= 0, 1)$ is obtained. We demonstrate that the precision of the estimation can be improved (with a larger Fisher information) by incorporating the correlation data, and we clarify in particular that the present sequential scheme can provide us with better estimation than a standard strategy, when only weak measurements can be performed.

[1] M. Guţă, Phys. Rev. A **83**, 062324 (2011).