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Second law of the information thermodynamics with entanglement transfer

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We present a new inequality which holds in the thermodynamical processes with measurement and feedback controls with using only the Helmholtz free energy and the entanglement of formation[1]:

$$W_{\text{ext}} \leq -\Delta F - k_B T \Delta E_F^{SB-R}, \tag{1}$$

where E_F^{SB-R} is the entanglement of formation [2] between system and baths SB and the reference system R, and $-k_B T \Delta E_F^{SB-R}$ is always positive. The difference ΔE_F^{SB-R} is taken between before and after the unitary interaction U_{SP} between the system S and the probe P during the measurement:

$$-\Delta E_F^{SB\text{-}R} = E_F^{SB\text{-}R}|_{\text{before } U_{SP}} - E_F^{SB\text{-}R}|_{\text{after } U_{SP}}. \tag{2}$$

It is easier to achieve the upper bound in the new inequality (1) than in the Sagawa-Ueda inequality[3]:

$$W_{\text{ext}} \leq -\Delta F + k_B T I_{\text{QC}}. \tag{3}$$

The new inequality has clear physical meaning: the quantity $-\Delta E_F^{SB-R}$ is also the amount of entanglement transfer from the system S to the probe P through the interaction U_{SP}. In the above thermodynamical processes, the work which we can extract from the thermodynamic system is greater than the upper bound in the conventional thermodynamics by the amount of the entanglement extracted by the unitary interaction of the measurement. In other words, from a thermodynamical point of view, we can interpret the entanglement transfer as the information transfer.

[1] H. Tajima, arXiv:1212.0407.
[2] Charles H. Bennett, David P. DiVincenzo, John A. Smolin, and William K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[3] T. Sagawa and M. Ueda, Phys. Rev. Lett, **100** 080403 (2008).