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Extension of de Bruijn type identity for nonequilibrium dynamics

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An information-theoretical identity which is called de Bruijn identity has a link with the heat equation as a physical background. This identity tells us that the time derivative of information entropy equals to the Fisher information (e.g. [1]). Recently, the relative version of it was derived and has been called the de Bruijn type identity by researchers in Princeton University. It states that the time derivative of the relative entropy (KL divergence) equals to the relative Fisher information (RFI).

The roles of the relative Fisher information in nonequilibrium processes are not necessarily investigated in full in the literature. We have reported a profound link with dissipated work during nonequilibrium processes at the previous YSM-SPIP meeting [2]. We show here that the RFI is replaced by the corresponding information-theoretical quantities, in the reflection of the nature of nonequilibrium dynamics [3].

In this study, in order to explore the analogue (or extension) of de Bruijn type identity for a wide class of nonequilibrium dynamics, we reformulate it for systems with probabilistic currents in the context of statistical physics. To be precise, the time derivatives of relative entropies under the continuity equation are presented, which shows that the conservation of distance between a pair of distributions is not guaranteed except for Liouville evolutions. As a specific case, we mention a general nonlinear Fokker-Planck equation.

These formulations can provide a physical interpretation of the de Bruijn type identity that holds for the system other than a heat equation. These investigations strongly encourage us to further explore links between information-theoretic quantities and nonequilibrium physical laws.

[1] T. Cover and J. Thomas J., *Elements of Information Theory 2nd ed.*, Wiley-Interscience, (2006).
[2] T. Yamano, *Dissipated work in nonequilibrium process and relative Fisher information*, for the Proceedings of 4th YSM-SPIP held at Tohoku University (2013).
[3] T. Yamano, *de Bruijn type identity for systems with flux*, submitted to EPL (2013).