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Frequency Adaptation in Directed Networks

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Complex networks of interacting oscillators with adapting frequency differs and complements Christiaan HUYGENS' well studied topic of synchronized dynamics. Synchronisation expresses a notion of "elasticity": it enables oscillators to produce common dynamical patterns thanks to the continuous action of mutual interactions. However, each oscillator recovers its individual frequency when interactions between them are removed. On the other hand, adaptation exhibits an idea of "plasticity": it enables the formation of permanent common dynamical patterns that alter the individual oscillators even after interactions are removed.

We here consider a collection of phase oscillators coupled via a directed network (i.e. the underlying adjacency matrix is not symmetric). Contrary to the usual practice where the individual frequencies are fixed, we here allow them to adapt. To do so, we introduce in each local oscillator an additional degree of freedom which allows individual frequencies to be time-dependent with their own dynamics, called adaptive mechanisms. Therefore, individual frequencies acquire the status of variables of the system. The adaptive mechanisms dependent only on the phases, and, according to the system's state, tune the value of the individual frequencies.

Together with the usual phase coupling, the adaptive mechanisms allow the oscillators to converged towards a consensual state, where all phases and frequencies are in synchronous. If the system converges and once this consensual state is reached, it subsists even if interactions are removed (plastic behavior). In the already studied model with undirected networks (i.e. symmetric adjacency matrix), the asymptotic value of the consensual frequency towards which all oscillators convert to, is analytically determined. Here, even for an undirected network (i.e. symmetric assumption on the adjacency matrix is relaxed), we are still able to analytically express the consensual frequency. For a class of adaptive mechanisms, the value of the consensual frequency depends on the initial phases and frequencies values and on the adjanceny matrix. This is to be compared with the case of undirected network, where the consensual frequency depends only on the initial values of the individual frequencies.

The dynamical system here studied can be interpreted as the deterministic version of a 1) KURAMOTO type modelling of neuron network, 2) network of coupled insurances/reinsurances or 3) collection of coupled stocks prices from the financial markets.