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Relation between Entropy Production and Path Probability in the Presence of Hidden Degrees of Freedom

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Detailed fluctuation theorem (DFT) states that the entropy production in the heat baths is equal to the logarithmic ratio of the forward and backward path probabilities [1]. By assuming this DFT, one may theoretically investigate novel thermodynamic relations in the context of stochastic processes, or estimate the heat exchanged between the system and the environment from stochastic trajectories observed in mesoscopic experiments.

Although DFT has been derived in the several systems (e.g. Hamiltonian system weakly coupled to heat baths [2]), it has recently become clear that DFT does not always hold, in the sense that the "entropy production", formally defined as the logarithmic ratio of the path probabilities, may deviate from the physical entropy production [3,4]. The essence of this problem lies in the fact that the "entropy production" may change its value depending on the scale of stochastic description. In such cases, DFT does not hold at the experimentally observed scale, even if it is justified at the microscopic level.

We approach this problem by considering the excess entropy production, which is a key quantity in steady state thermodynamics [5]. We show that the sum of the so-called excess entropy production derived from the "entropy production" and the entropy increment of the system, is essentially kept invariant with respect to the change in the scales of description [6]. This invariance suggests that the formal "entropy production" will always reflect the physical excess entropy production in the total system correctly.

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