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A Work Relation for NESS that involves the Violation of Linear Response Relation — With an Application to Adiabatic Pumping

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We extend Jarzynski's work relation and the second law of thermodynamics to a heat conducting system which is operated by an external agent. Suppose the system is operated via a parameter ν and driven by two heat baths. We finx their inverse temperatures as $\bar{\beta}$ in mean and $\Delta\beta$ in difference. The Jarzynski equality is extended as

$$\left\langle e^{-\bar{\beta}(W-\Delta F)}\right\rangle^{\hat{\nu}} = \left\langle e^{-\Delta\beta\, Q_{\mathrm{t}}}\right\rangle_{\mathrm{m}}^{\hat{\nu}^\dagger}, \tag{1}$$

where $\hat{\nu}^\dagger$ is the reverse protocol of a protocol $\hat{\nu}$ defined in $t \in [-\tau, \tau]$ and Q_{t} is the heat transfer between two heat baths. In the quasi-static limit, (1) is written as

$$\left\langle W\right\rangle^{\hat{\nu}} = \Delta F - \frac{\Delta\beta}{\bar{\beta}}\int_{-\tau_{\mathrm{s}}}^{\tau_{\mathrm{s}}} dt J_{\mathrm{viol}}^{\hat{\nu}}(t) + O(\epsilon^3), \tag{2}$$

which is a relation corresponding to the Gibbs relation in equilibrium thermodynamics. A new nonequilibrium contribution in the right hand of (1) is expressed as the violation of the (linear) response relation in (2), where

$$J_{\mathrm{viol}}^{\hat{\nu}}(t) \quad = \quad \left\langle J(t)\right\rangle^{\hat{\nu}} - \frac{\Delta\beta}{2}\int_{-\infty}^{\infty} ds\left\langle J(t); J(s)\right\rangle^{\hat{\nu}}, \quad (-\tau \le t \le \tau), \tag{3}$$

which is caused by the operation and vanishing in the steady state (corresponding to the linear response relation).

Remarkably, the obtained work relation explores a new understanding to an adiabatic pumping. Up to the first order in $\Delta\beta$, (2) is written as

$$\left\langle W\right\rangle^{\hat{\nu}} = \Delta F - \frac{\Delta\beta}{\bar{\beta}}\left\langle Q_{\mathrm{t}}\right\rangle^{\hat{\nu}_{\mathrm{eq}}} + O(\epsilon^2), \tag{4}$$

where $\langle Q_{\mathrm{t}}\rangle^{\hat{\nu}_{\mathrm{eq}}}$ is a pumping current produced by the external operation in equilibrium. Remembering the conventional thermodynamic relation, the adiabatic operational work satisfies $W = 0$ in the cyclic protocol, even when it produces pumping current. This is simply due to the translational invariance in equilibrium. On the other hand, (4) indicates that the pumping current becomes involved in thermodynamics by the extension to nonequilibrium.

By utilizing (4), we find that the adiabatic pumping in equilibrium is connected to the probability density in NESS. We define a pumping density expressing the ability of pumping in an operational parameter space. Once we get it, we can predict the amount of the pumping current for every designed protocol. We demonstrate numerical results of the pumping density for flashing ratchet models.

[1] N. Nakagawa, *Phys. Rev.* E **85**, 051115 (2012), arxiv:1109.1374