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Integral quantum fluctuation theorems under measurement and feedback control

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Fluctuation theorems and Jarzynski equalities have attracted considerable interest, since they give equalities on physical quantities in nonequilibrium processes, in contrast with the conventional second law, which gives inequalities. Due to the recent advances in experimental techniques, controlling small fluctuating systems based on a measurement and feedback has also attracted much interest in the field of nonequilibrium statistical mechanics. It has been shown that for a quantum system, the conventional second law should be modified by including the mutual information content when we consider measurement and feedback control [1,2]:

$$\langle \sigma^S \rangle \geq -\langle I \rangle, \quad \langle \sigma^M \rangle \geq \langle I \rangle, \tag{1}$$

where σ^S and σ^M are the entropy production of the system and the memory, respectively, and $\langle I \rangle$ is the quantum-classical (QC) mutual information content [1]:

$$\langle I \rangle =: S(\rho_{\mathrm{i}}^S) - \sum_k p_k S(\rho^S(k)), \tag{2}$$

where ρ_{i}^S is the pre-measurement state, $\rho^S(k)$ is the post-measurement state conditioned on the measurement outcome k, and $S(\rho)$ is the von Neumann entropy. Equation (2) measures the acquired knowledge of the system due to the measurement since it gives the reduction of an uncertainty of the system (expressed by the von Neumann entropy) when a measurement is performed. Associated with the modification of the second law, integral fluctuation theorems are also modified by including the mutual information content. For a classical system, the integral fluctuation theorem under general information processing was obtained [3], and its quantum version was discussed in Ref. [4]. However, the authors in Ref. [4] consider a projective measurement followed by a classical error, which does not include a general quantum measurement described by a measurement operator M_k, and thus the obtained equality does not reproduce the inequalities (1).

In this presentation, we show the following quantum integral fluctuation theorems for a feedback controlled system and a memory:

$$\langle \mathrm{e}^{-\sigma^S - I} \rangle = 1, \quad \langle \mathrm{e}^{-\sigma^M + I} \rangle = 1, \tag{3}$$

where I is given by the (unaveraged) QC-mutual information. The modified version of the equalities include the QC-mutual information content, which expresses the information exchange between the system and the memory. When we take the first cumulant of Eq. (3), the generalized second laws under measurement and feedback control (1) are reproduced.

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[3] T. Sagawa and M. Ueda, Phys. Rev. Lett. **109**, 180602 (2012).
[4] M. Morikuni and H. Tasaki, J. Stat. Phys. **143**, 1 (2011).